



## The physics of variable area flow meters

The flow rate of gases and liquids can be easily and relatively accurately determined using variable area flow meters. A tapered cone is fitted vertically with flow from bottom to top. The upward flowing medium causes the float to rise until the annular gap between float and tube wall is so large that the forces acting on the float are balanced and the float has attained its floating state.

Three forces act on the float (Fig. 1). The force due to weight  $F_G$  is directed downwards:

$$F_G = V_S \cdot \rho_S \cdot g \quad [1]$$

Two forces act upwards:  
buoyancy  $F_A$ :

$$F_A = V_S \cdot \rho_m \cdot g \quad [2]$$

and drag  $F_S$ :

$$F_S = c_w \cdot A_S \cdot \frac{\rho_m \cdot v^2}{2} \quad [3]$$

The following applies to the state of equilibrium, or floating state:

$$F_G = F_A + F_S \quad [4]$$

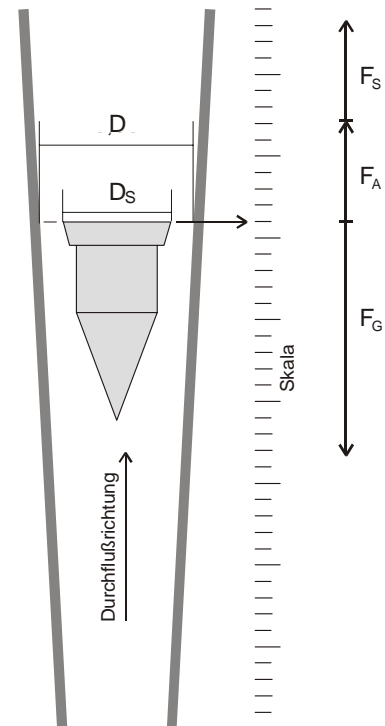
The flow rate is:

$$q_v = v \cdot A = v \cdot \frac{\pi}{4} \cdot (D_k^2 - D_s^2) \quad [5]$$

Drag coefficient  $c_w$  is transferred into the flow coefficient.

$$\alpha = \sqrt{\frac{1}{c_w}} \quad [6]$$

The flow coefficient  $\alpha$  is dependent on the geometric shape of the measuring tube and float, and also includes the friction factor. As an empirical value,  $\alpha$  appears in device-related





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characteristics that are included in the calculation. These characteristics also include the Ruppel factor, which allows inclusion of the product-related parameters viscosity and density.

Considering the above equations, it is possible to form the general flow equation for variable area flow meters.

Volume rate of flow:

$$q_v = \frac{\alpha}{\rho_m} D_s \sqrt{g \cdot m_s \cdot \rho_m \left(1 - \frac{\rho_m}{\rho_s}\right)} \quad [7]$$

Mass rate of flow:

$$q_m = \alpha \cdot D_s \sqrt{g \cdot m_s \cdot \rho_m \left(1 - \frac{\rho_m}{\rho_s}\right)} \quad [8]$$

Due to the taper of the measuring tube, the size of the annular gap will change according to the vertical position of the float. Thus, the float position supplies information on the flow rate. When a glass measuring tube is used, therefore, the measured value can be read direct from the scale attached to the measuring tube.

Compared to the differential-pressure method, e.g. at orifice plates or in the venturi tube, there is a physical analogy: The method of calculation is similar. The significant difference can be described as follows: In the differential-pressure method of measurement the cross-sectional flow area remains constant and the differential pressure changes with the flow rate, whereas in the float principle the cross-sectional area changes and the pressure difference remains the same.

- $V_S$ : Volume of float
- $A$ : Annular gap area
- $A_S$ : Cross-sectional area of float at the reading line
- $m_S$ : Mass of float
- $\rho_S$ : Density of float
- $\rho_m$ : Density of liquid product
- $c_W$ : Drag coefficient
- $v$ : Flow velocity of liquid product
- $D_k$ : Inside diameter of cone at reading line
- $D_S$ : Diameter of float at reading line